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Optimal investment and finance in renewable resource harvesting

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Abstract

The paper studies a dynamic optimization problem in renewable resource harvesting, capital investment, and financing. It combines two different approaches, viz., renewable resource harvesting with capital investments, and investment policy under a borrowing/lending constraint according to which the interest rate on outstanding debt/lending increases with cumulative debt/lending. The problem is set up as an optimal control model, having two state variables (resource stock and stock of equity) and two controls (effort rate and dividend payout rate). The solution is identified by the maximum principle and a synthesizing procedure.

Key words: Investment and financing policies; Renewable resource; Optimal control
JEL classification: D21; G31; G32; Q22

1. Introduction

Bioeconomic theory tells us that in an open-access fishery rents tend to dissipate and stocks may be extincted. These findings have support in real life. To prevent overfishing and possible depletion of stocks, considerable efforts have been devoted to the study and development of appropriate means of regulation, in the literature and in the design of real-life fishery regulation

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policies. Among a wide array of instruments, one means of regulation is to give sole-owner rights to a single firm (or a cartel). By defining clearly the property rights to the fishery, incentives and responsibility for sustainable resource use may be established. Trends of converting common property fisheries into single-owner fisheries have been seen in Canada, New Zealand, Finland, and Iceland (see Novak et al., 1995).

This paper characterizes optimal dynamic harvesting, capital investment, and financing policies of a single owner fishery. To our best knowledge, the paper is the first attempt to combine two different streams of research: one dealing with a firm's optimal dynamic decisions on capital investments and financial structure, the other one being concerned with optimal harvesting of a renewable resource and capital investments in harvesting effort capacity. Hochman et al. (1973), Schworm (1980), and Steigum (1983) studied capital investments under financial constraints but did not include resource harvesting. Clark (1976) and Clark et al. (1979) studied resource harvesting and capital investment. The assumptions on the firm's financial structure were rather extreme: either the firm was entirely self-financed or had unlimited access to external financing in a perfect capital market.

This paper introduces a more balanced financial assumption. We allow the firm to borrow or lend in an imperfect capital market such that the interest rate increases with the firm's cumulative amount of debt/lending. This represents a generalization of the work of Clark (1976) and Clark et al. (1979). To keep the model tractable we need, however, to replace the assumption of irreversible investment in Clark et al. (1976) with an assumption of perfectly reversible investment. This is a simplification of the Clark et al. (1976) model. The simplification may not be overly restrictive, since it turns out that the optimal investment rate is never negative, except at the initial instant of time and possibly, but not necessarily, on a finite interval of time during the final approach to the steady state.

The paper progresses as follows. Section 2 contains a brief description of the dynamic optimization model. The intention of this section is to give the reader the basic ingredients of our problem, without going into detailed motivations for the assumptions. Section 3 states our main result and its economic interpretations, but without any proofs. Section 4 and the appendices provide the rigorous solution of the optimal control problem, including all necessary proofs. Section 5 concludes and offers our comments on the assumptions. This section also discusses some modifications and extensions.

2. The model

This section states the dynamics that describe the resource harvesting, capital investment, and finance. It also identifies the firm's objective. To set up the model, we need a number of assumptions. The present section simply states our assumptions. In Section 5 we discuss these hypotheses and point to some modifications and extensions.

2.1. Resource harvesting dynamics

A firm operates a single-species commercial fishery to which the firm has been granted exclusive fishing rights. For $t \in [0, \infty)$, let $S = S(t)$ denote the stock of resource by time t and let $E = E(t)$ represent the firm's effort rate at time t. The natural growth of the resource is given by a function $F(S) \in C^2$ which satisfies the standard assumptions

$$F(S) > 0 \quad \text{for} \quad S \in (0, S_m), \quad F(0) = F(S_m) = 0,$$

$$F(S) < 0 \quad \text{for} \quad S > S_m, \quad F''(S) < 0 \quad \text{for} \quad S > 0. \tag{2.1}$$

The stock of resource evolves according to the Gordon–Schaefer model

$$\dot{S}(t) = F(S(t)) - qE(t)S(t), \qquad S(0) = S_0 \in (0, S_m), \tag{2.2}$$

in which $q = \text{const.} > 0$ is the 'catchability' coefficient. The firm sells its harvest at a constant unit price $p > 0$. Revenues from harvesting are $pqES$. The operating cost is cE, in which $c = \text{const.} > 0$ represents the operating cost per unit of effort. The profit from harvesting equals $(pqS - c)E$ at time t.

2.2. Capital stock dynamics

Let $K = K(t)$ represent the stock of real capital available by time t for the firm's harvesting operations. K can be interpreted to represent the number of standardized fishing vessels available at time t. The purchase price of a unit of capital (the capacity cost) is constant and has been normalized to one. Let $I = I(t)$ denote the firm's gross investment rate at time t and let $a = \text{const.} > 0$ be the depreciation rate of capital. The equation of motion of K is standard:

$$\dot{K} = I(t) - aK(t), \qquad K(0) = K_0 > 0. \tag{2.3}$$

It is sensible to introduce an upper bound, $E_{\max}$ say, on harvesting effort. Let this upper bound be determined by the stock of productive capacity, i.e., the number of fishing vessels available. Thus, we take $E_{\max}$ equal to $K(t)$. Impose the following constraint:

$$0 \le E(t) \le E_{\max} = K(t), \qquad \forall\, t \in [0, \infty). \tag{2.4}$$

We assume investment $I(t)$ to be perfectly reversible, making disinvestment possible. Moreover, there are no bounds on either investment or disinvestment

$$-\infty \leq I(t) \leq +\infty. \tag{2.5}$$

The equality signs in (2.5) admit any amount of capital goods to be purchased or sold instantaneously, that is, jump increases or decreases in K are feasible. An important implication of (2.5) is the following. Since any amount of capital can be sold immediately (at its purchase price), *an optimal solution will not involve excess capacity*, i.e., for all t it holds that $E(t) = K(t)$.[1] For, if $E(t)$ were smaller than $K(t)$, the firm would have excess capacity, generating no revenues, and the firm would be better off to sell such idle capacity. Due to (2.5) a sale can be effected instantaneously. It would yield a lump sum income $K - E > 0$ that would be used to reduce debt/increase lending. The upshot is that an optimal E always equals its upper limit, except possibly at the isolated instant $t = 0$:

$$E(t) = E_{\max} = K(t). \tag{2.6}$$

Technically, (2.6) implies that K is no longer needed as a state variable, and both operating and capacity costs can be treated as variable costs.

2.3. Financial dynamics

A main purpose of the paper is to introduce a novel feature, the possibility of borrowing and lending, into the above resource harvesting and capital investment model. The firm has access to two means of finance. First, it is not obliged to distribute all its cash flow to the owners and may choose to finance its operations and investments, in part or in total, by retained profits. Second, the firm may borrow to obtain additional funds. The firm may also choose to lend, that is, to hold financial assets. For simplicity, we shall disregard the possibility of simultaneous borrowing and lending. Define $X = X(t)$ as the stock of equity by time t, and let $B = B(t)$ represent the level of debt/lending by time t. Then $B > 0$ means the firm has debt (but does not lend), whereas $B < 0$ would mean that the firm lends (but has no debt). The accounting identity of the balance sheet must be satisfied.

$$K = X + B. \tag{2.7}$$

The firm acts under certainty and employs a discount rate, i, being constant and independent of the firm's capital structure. For the lenders, however, the

[1] See also Clark et al. (1979), Kennedy (1989).

prospects of the firm are uncertain. This is modelled by supposing that lenders demand a default risk premium. If the firm has a high amount of outstanding debt, a higher interest rate will be charged than if debt were smaller (cf. Hochman et al., 1973). Define the interest rate function $r(B)$ as follows:

$$r(B) = r_o \quad \text{if} \quad B \leq 0, \qquad r(B) = r_o + kB \quad \text{if} \quad B > 0, \tag{2.8}$$

where r_0 and k are positive constants. The firm receives a constant interest rate on any amount lent and pays an interest cost being proportional to the amount borrowed.[2] Define the 'cost of funds' as follows:

$$C(B) = r_0 B \quad \text{if} \quad B \leq 0, \qquad C(B) = r_0 B + kB^2 \quad \text{if} \quad B > 0. \tag{2.9}$$

If B is negative, the amount $r_0 B$ represents an interest income. The marginal cost of funds will play an important role in our developments: notice that the marginal cost function $C'(B)$ is piecewise linear and has a kink at $B = 0$.

Let $D = D(t)$ be the rate of dividend payout, and introduce the constraint

$$0 \leq D(t) \leq D_{\max}. \tag{2.10}$$

$D_{\max}$ is an artificial upper bound, imposed because the optimal control problem will turn out to be linear in D. We shall see, however, that $D_{\max}$ has no significance. A negative dividend rate can be interpreted to mean an issue of new shares. We have assumed that issuing new shares is not an option of the firm, except possibly at time zero. Thus, the left-hand inequality in (2.10) is imposed for all $t > 0$. At time 0, any $D < 0$ is permitted. This gives the firm the possibility, through an impulsive issue of new equity, to make an instantaneous adjustment (an increase) of its initial stock of equity.

From the firm's profit and loss account the following equation can be derived

$$[pqS(t) - c]E(t) - aK(t) - C(B(t)) = D(t) + \dot{X}(t), \tag{2.11}$$

which simply states that net accounting profits (the term on the left-hand side) are either paid out as dividends or are retained.[3]

[2] We have chosen a linear interest function but any increasing function of B would do.

[3] It may be sensible to constrain D to be less than the left-hand side of (2.11). This would prevent the firm from borrowing merely to pay dividends. (Such a practice is, in some countries, illegal.) Note that such a constraint would be equivalent to X is nondecreasing. We disregard the constraint: it will turn out that it is satisfied by the optimal solution.

The firm has no incentive to hold cash which means that the cash account exhibits the identity

$$[pqS(t) - c]E(t) = D(t) + C(B(t)) - \dot{B}(t) + I(t), \tag{2.12}$$

which simply states that cash inflow equals cash outflow at all points of time. Operating income (the left-hand side) is used to pay dividends, interest on debt, to redeem debt (if $B > 0$), and the gross investment expenditure. The cash flow to the owners equals $D(t)$.

We impose the following assumption on the relationship between interest rates r_o and i:

Assumption 1. $r_o < i$.

The assumption is motivated by the following. A solution with $i < r_o$ would imply a long-run accumulation of financial assets, that is, $B \to -\infty$ as time tends to infinity.[4] If $i < r_o$, the firm can lend an additional dollar of earnings – instead of paying it out as dividends – and earn a rate of return of r_o. The owners would favor this. If $i = r_o$, the owners would be indifferent between having a dollar of dividends and letting the firm retain (and lend) the dollar. This situation would prevail in a perfect capital market. One might wonder, however, if family-owned fishery firms operate under such ideal conditions. Technically, $i = r_o$ would imply that the steady state value of B is indeterminate.

2.4. The firm's objective

The firm's objective is that of the owners. It is the present value of the dividend stream over an infinite horizon, starting at time 0,

$$J = \int_0^\infty e^{-it} D(t)\, dt. \tag{2.13}$$

Inserting dividends D from (2.12) into (2.13) yields

$$J = \int_0^\infty e^{-it}[(pqS(t) - c)E(t) - I(t) + \dot{B}(t) - C(B(t))]dt, \tag{2.14}$$

[4] See also Steigum (1983), Hochman et al. (1973).

which shows that the objective J represents the discounted net cash flow. Using (2.3) and (2.6) yields, whenever E is differentiable,

$$\dot{E}(t) + aE(t) = I(t). \tag{2.15}$$

Finally, substituting the left-hand side of (2.15) into (2.14), and integrating by parts, yields

$$J = \int_0^\infty e^{-it}[pqS(t) - c - a - i]E(t)\mathrm{d}t + \int_0^\infty e^{-it}[i - r(B)]B(t)\mathrm{d}t + X_0. \tag{2.16}$$

The first integral on the right-hand side (2.16) is the objective employed by Clark et al. (1979, Sect. 3). Hence, their objective is a special case of (2.16), occurring for $B \equiv 0$.

3. The main results

This section presents, without any proofs, the main results and their economic interpretations. For the proofs and more details of the optimal solution, we refer the reader to Section 4 and the Appendices. We omit from now on, when no confusion can arise, the time argument t, and start by summarizing the optimal control problem of the firm. The firm wishes to choose its controls E and D so as to maximize the objective J, subject to

$$0 < D \leq D_{\max}, \qquad 0 \leq E,$$

$$B = E - X,$$

$$\dot{S} = F(S) - qES, \qquad S(0) = S_0 \in (0, S_m),$$

$$\dot{X} = (pqS - c - a)E - C(B) - D, \qquad X(0) = X_0 > 0.$$

The state constraints $X, S \geq 0$ must be satisfied for all t. In this section, we disregard these constraints, but Section 4 shows that $X > 0$ and $S > 0$ are satisfied in the optimal solution.

The model has two states (S, X), two controls (E, D), and exhibits certain linearities. Models of this kind are known to yield relatively complicated optimal trajectories, involving – among other things – nontrivial switches of the controls. This will also be the case in the present model. However, since the model is quite concrete, the bio-economic reasons for the optimal harvesting and shareholder consumption policies should be transparent. The same is true for the optimal capital investment and financing policies.

The nonnegativity conditions on the two controls imply the existence of four different candidate paths. We briefly describe these paths. More details can be found in Section 4, in which we also show infeasibility of the path $E = 0$, $D > 0$.

3.1. The candidate paths

Path 1: $E = D = 0$. Since E equals zero, B is negative, that is, the firm lends. The identity $E = K$ shows that the firm has no capital goods at all. Moreover, investment I is zero. Hence, on this path, the firm is out of the harvesting business and all its equity is invested in financial assets. The stock of equity, X, increases by the interest income from lending, $r_0 X$. No dividends are paid out. The amount lent increases at the same rate as equity. In the absence of any harvesting effort, the stock of resource increases.

Path 2: $D > 0$, $E > 0$. The firm has debt which, however, remains at a constant level, B_i say. B_i is determined by the intuitive condition $C'(B_i) = i$; the marginal cost of funds must equal the time preference rate. The level of debt is given by $B_i = (i - r_0)/2k$ which is positive by Assumption 1. Small values of r_0 and/or k induce the firm to hold a larger level of debt.[5] The resource stock stays at a constant level, S_i say, which is implicitly given by

$$F'(S_i) + \frac{F(S_i)(c + a + i)}{S_i[pqS_i - (c + a + i)]} = i. \tag{3.1}$$

To maintain the resource stock at the level S_i, an effort rate of $E_i = F(S_i)/qS_i = \text{const.} > 0$ is needed. The gross investment rate, needed to maintain effort capacity E_i, is given by $aE_i > 0$. The level of equity is constant and equals $X_i = E_i - B_i$. Dividends are positive and constant, given by $D_i = (pqS_i - c - a)E_i - C(B_i)$. It holds that $D_i > iX_i$, which is intuitive: the dividend payout should exceed the income, iX_i, that could have been earned if the owners choose to liquidate the firm at the instant of time at which Path 2 starts.

Path 3: $D = 0$, $E > 0$. This path has two subcases. *Path 3a* has $B \leq 0 \leftrightarrow E \leq 0 \leq X$, and *Path 3b* has $B > 0 \leftrightarrow E > X$. Path 3a is a singular path, associated with a linear part of the Hamiltonian, which occurs for $B \leq 0$. Path

[5] Recall that r_0 is the fixed unit interest cost, kB the variable unit interest cost.

3a admits a constant stock level, S_r say. Path 3a is, however, a 'degenerate' path because, in an optimal solution, Path 3a only occurs at an isolated point of time. On Path 3b the following condition holds:

$$F'(S) + \frac{F(S)[c + a + C'(B)]}{S[pqS - (c + a + C'(B))]} - \frac{C''(B)\dot{B}}{[pqS - (c + a + C'(B))]} = C'(B). \quad (3.2)$$

The stock level satisfying (3.2) depends on both B and its time derivative. Denote this level by

$$S_{\mathrm{FB}} = S(B, \dot{B}). \quad (3.3)$$

The meaning of the subscript 'FB' will be clear later on. On Path 3b, the optimal effort rate is the unique solution, E^* say, of the following equation:

$$(1 + \lambda_2)[pqS - (c + a + C'(E^* - X))] - \lambda_1 qS = 0,$$

where λ_1 and λ_2 are the shadow prices (costate variables) of the resource stock and the stock of equity, respectively. Implicit differentiation yields a characterization of E^*. The derivative $\partial E^*/\partial S = q[(1 + \lambda_2)p - \lambda_1]/(1 + \lambda_2)C''(B)$ is positive which means *ceteris paribus* that the larger the stock of resource, the larger the effort rate.[6] It holds that $\partial E^*/\partial X = 1$, which means that a marginal dollar added to equity is used for an equivalent increase in the capital stock K. (Recall that the price of a unit of capital stock equals one.) The derivative $\partial E^*/\partial \lambda_1 = -qS/(1 + \lambda_2)C''(B)$ is negative. Thus, when the resource becomes more valuable to the firm, effort should be decreased which, in turn, decreases the reduction of the stock of resource. Finally, $\partial E^*/\partial \lambda_2 = \lambda_1 qS/(1 + \lambda_2)^2 C''(B) > 0$. An increase in the shadow price of equity means that the stock of equity becomes more valuable to the firm. The firm is induced to raise its revenue which, in turn, requires an increase in effort. In the balance sheet (cf. (2.7)), X increases on the right-hand side, and $E(= K)$ increases accordingly on the left-hand side.

Finally, on Path 3b it holds that equity X is increasing and gross investment I is strictly positive on an initial as well as a final interval of time. On an intermediate interval of time along Path 3b, disinvestment may, but does not necessarily, occur.

[6] See also Jørgensen and Sorger (1990), Seierstad and Sydsæter (1987, p. 263).

3.2. The optimal harvesting effort and dividend policies

Having provided a brief characterization of the three candidate paths, we turn to the problem of synthesizing these paths. Applying a sufficiency theorem we verify in Appendix 3 the optimality of the solution.

We limit our analysis to cases in which the initial stock of resource is relatively small. Such situations may occur if excessive open-access harvesting had been going on, leading to overexploitation of the resource. As of time 0, the resource is placed under the firm's management.[7] To be specific, we study cases where $S_0 < S_i$. Further, we confine our interest to cases in which the firm initially is 'small': $X_0 < X_i$.[8] Recall that S_i and X_i are the steady state values of S and X, respectively, along Path 2.

We now construct the optimal solution as the optimal sequence of the candidate paths. The following proposition is a main result of the paper. To state the proposition, recall that Path 3a can only occur at an isolated instant of time. Denote this instant by τ. Moreover, with reference to (3.3), define the following level of resource stock:

$$S_{\mathrm{F0}} = S(0, \dot{B}(\tau^+)). \tag{3.4}$$

Proposition 1. (i) *For any S_0 such that $0 < S_0 < S_{\mathrm{F0}}$, the sequence Path 1 → Path 3a → Path 3b → Path 2 is optimal.* (ii) *For any S_0 such that $S_{\mathrm{F0}} < S_0 < S_i$, the sequence Path 3b → Path 2 is optimal.*

Fig. 1 depicts qualitatively the evolution over time of the states and the control variables. The dotted segments of trajectories E and B on Path 3b are used to indicate that this behavior is conjectured: we have been unable to provide a formal proof but the behavior was supported by numerical simulations with a logistic growth function.

Let us interpret the optimal trajectory in the first part of the proposition. This result states the following. If the firm observes that the initial stock of resource S_0 is less than the threshold level S_{F0}, zero harvesting effort is initially optimal and should be continued as long as the current stock $S(t)$ does not exceed the level S_{F0}. Thus, the fishery should be shut down at time 0 and not resumed until the resource stock has grown sufficiently, that is, has reached the level S_{F0}. Thus, when the resource stock is below the threshold, the fish population is 'too small' to make harvesting profitable. This is because the cost of fishing is higher, the lower the stock of resource.

[7] See Clark et al. (1979), Kennedy (1989), Novak et al. (1995).

[8] If $X_0 > X_i$, an impulsive payout of dividends can be used to reduce X_0 to any level below X_i, but note that an $S_0 > S_i$ cannot be reduced impulsively (because E is upper bounded).

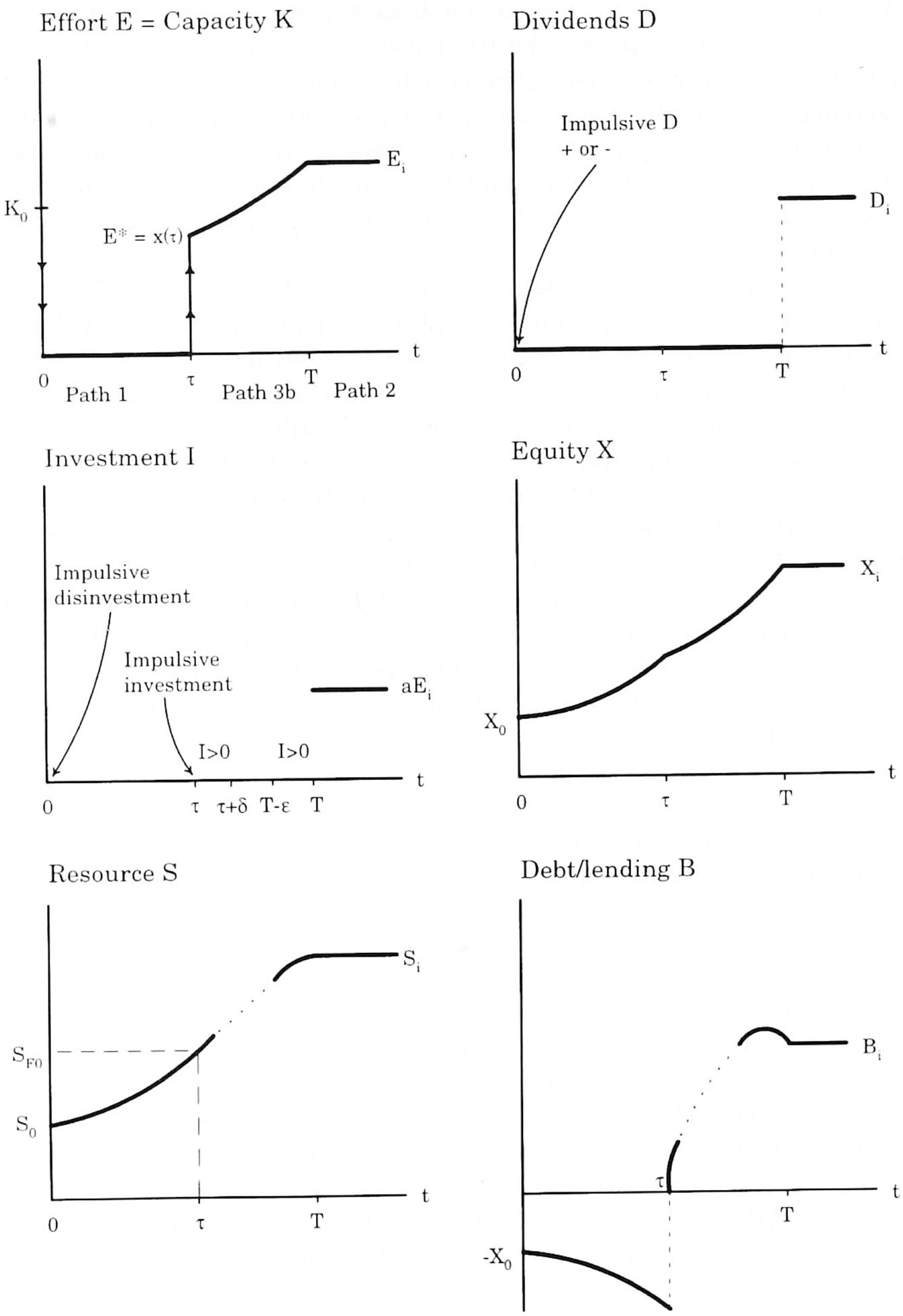
Effort E = Capacity K
K_0
$E^* = x(\tau)$
E_i
t
0
τ
T
Path 1
Path 3b
Path 2
Dividends D
Impulsive D
+ or -
D_i
t
0
τ
T
Investment I
Impulsive
disinvestment
Impulsive
investment
aE_i
I>0
I>0
t
0
τ
$\tau+\delta$
T-ε
T
Equity X
X_i
X_0
t
0
τ
T
Resource S
S_i
S_{F0}
S_0
t
0
τ
T
Debt/lending B
B_i
τ
t
T
$-X_0$

Fig. 1

At time 0, $K_0 = X_0 + B_0$ is given and, since K_0 and X_0 are fixed initial values of the state variables, B_0 will be fixed too. Note that $K_0 > B_0$. As long as harvesting is unprofitable, there is no need for capacity. Hence, at $t = 0$, the firm should make an impulsive disinvestment so as to reduce K_0 to zero. This means that the entire initial capital stock is sold, generating the lump sum income K_0. If B_0 happens to be positive, the amount K_0 is used to reduce debt to zero and the remaining amount, $K_0 - B_0$, will be lent. If B_0 is negative, the whole amount K_0 is lent.

After these initial transactions, Path 1 is applied for $t \in (0, \tau)$. During this interval of time, harvesting effort, capital investment, and capital stock are all equal to zero. All the firm's equity is lent, that is, we have $B = -X < 0$. The stock of equity and the amount lent both increase exponentially on Path 1. No dividends are paid out. Since there is no harvesting, the population immediately starts to increase and continues to do so as long as Path 1 is followed.

At time $t = \tau$, the threshold level S_{F0} eventually is reached. The significance of this level is the following. When the stock $S(t)$ passes this level, it becomes rational for the firm to start fishing (subscript 'F' in S_{F0}). At time τ, the zero-effort Path 1 passes into the positive-effort Path 3b. This transition takes place through the degenerate Path 3a, which occurs only for $t = \tau$. Financially speaking, at time τ the firm sells all its financial assets which causes the variable B to jump from $B < 0$ to $B = 0$ (subscript '0' in S_{F0}). This transaction is made in order to be able to finance the instantaneous investment in capital goods needed to start up the harvesting operations.

Immediately after time τ, the firm – which is now on Path 3b – starts to attract debt as an additional means of finance. The reason is that retained earnings are insufficient to finance the capacity which is necessary for harvesting.[9] On Path 3b, the harvesting effort is strictly positive but not larger than to allow the population eventually to increase from the threshold S_{F0} to the steady state level S_i.[10] The latter is reached at time T. No dividends are paid out on Path 3b and, consequently, the stock of equity X increases on this path.

When the resource stock equals S_i at time T, the firm has reached steady state Path 2. In this stationary state, the firm invests at a constant and positive rate to maintain effort (and hence capital stock) at the constant level E_i. The remaining cash flow is distributed to the owners who now enjoy a positive dividend payout. The firm holds a positive and constant amount of debt, B_i, satisfying the condition $C'(B_i) = i$. The steady state levels of capacity and resource stock are essentially the same as in Clark et al. (1979). In their paper, however, the steady

[9] The income stream from the financial assets has ceased and profits accruing from harvesting are still relatively small.

[10] A numerical example with a logistic growth function suggests that S is monotonically increasing on Path 3b.

state capacity level is reached by a final, impulsive investment. In our model, the transition into the steady state capacity level is smooth.

The steady state population level S_i admits an economic interpretation. Following Clark et al. (1979), let $c_{\mathrm{tot}}(S) = (c + a + i)/qS$ denote the unit harvesting cost and rewrite (3.1) as follows:

$$F'(S_i) - \frac{c'_{\mathrm{tot}}(S_i)F(S_i)}{p - c'_{\mathrm{tot}}(S_i)} = i. \tag{3.5}$$

The left-hand side of (3.5) is the marginal return on the stock. The first term is the marginal productivity of the stock, the second the marginal stock effect. The latter accounts for the sensitivity of the unit harvesting cost to the stock level. Hence, for $S = S_i$, the marginal return on the stock equals the discount rate i, and the owners are indifferent between investing one dollar in capacity and receiving the dollar as dividends.

Our solution implies that dividends are only paid out on the final interval (T, ∞). For all $t < T$, all profits are retained. This bang–bang policy may be somewhat extreme but is a consequence of D entering linearly in (2.13).[11] The policy is encountered in similar dynamic models of the firm (Van Hilten et al., 1993) and has been observed in growing industries (Judd and Petersen, 1986).

As already said, open-access resource harvesting tends to create efficiency problems. In real-life examples of modern fisheries, e.g., the North Sea herring fishery, a 'harvesting cycle' has been observed. Starting with relatively little harvesting, followed by a period of steadily increased harvesting, a climax is reached at which very substantial catches are taken. Such heavy exploitation of the population can lead to a collapse of fish stocks and, consequently, fishing activity. Our analysis suggests that converting a heavily exploited, open-access fishery into a single-owner enterprise may be a way to 'solve' the problems of overexploitation or extinction. Thus, facing initially an overexploited fish population, a rational owner would declare a fishing moratorium and, since he will not need his productive capacity for some time to come, he should dispose of that capacity. The proceeds can be invested in financial assets to earn an interest income. This scenario prevails until the fish population has recovered and it becomes profitable to begin harvesting. In practice, when harvesting goes on, we sometimes see very high harvesting rates, even leading to a collapse of the stock. Our model suggests that harvesting be carried out in a more 'sustainable' way in which the effort rate allows the resource stock to increase smoothly to reach its steady state level. Then harvesting should be maintained at a constant rate to keep the stock at its equilibrium level.

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[11] Using a nonlinear utility function of dividends would result in a smooth dividend policy; cf. Steigum (1983).

4. Solution of the optimal control problem

The optimal solution is identified by the necessary conditions of the maximum principle, extended by a synthesizing procedure. Then we verify that a sufficient condition for optimality, based on concavity, is satisfied. This solution procedure differs from the one employed by Clark et al. (1979) who 'guessed' a candidate solution and established optimality by a direct sufficiency condition.

In problems with an infinite horizon, the integral in the objective functional does not always converge. However, if the integrand of (2.13) is a bounded function and the rate of time preference is positive, the integral is defined. These conditions are fulfilled in our model, due to (2.10) and $i > 0$. We shall use the objective in the following, alternative form, which is easily obtained from (2.16):

$$J = \int_0^\infty e^{-it}[(pqS(t) - c - a)E(t) - C(B(t)) - iX(t)]dt.$$

Define the current-value Hamiltonian $H = H(E, D, S, X, \lambda_0, \lambda_1, \lambda_2)$ as follows:

$$H = \lambda_0[pqS - c - a)E - C(B) - iX] + \lambda_1[F(S) - qES]$$
$$+ \lambda_2[(pqS - c - a)E - C(B) - D].$$

Necessary optimality conditions require that controls E and D be piecewise continuous.[12] There must exist a constant $\lambda_0 \geq 0$ and continuous, piecewise continuously differentiable costates $\lambda_i(t)$ $[i = 1, 2]$ such that $\lambda_0, \lambda_1, \lambda_2$ do not vanish simultaneously for any t. We confine our interest to a solution with $\lambda_0 = 1$. The controls E and D must maximize the Hamiltonian at each instant of time. Except of points of discontinuity of (E, D), the following adjoint equations must be satisfied:

$$\dot{\lambda}_1 = i\lambda_1 - H_s = \lambda_1[i + qE - F'(S)] - (1 + \lambda_2)pqE, \tag{4.1}$$

$$\dot{\lambda}_2 = i\lambda_2 - H_x = (1 + \lambda_2)[i - C'(B)]. \tag{4.2}$$

State variables X and S must be continuous and piecewise differentiable and safisfy the constraints $X, S \geq 0$. We disregard the nonnegativity constraints but

[12] At points of discontinuity of E or D, the value of a control time function is taken as its left-hand limit.

shall show that they hold with strict inequality in the optimal solution. Some preliminary observations are in order.

(i) For $0 \leq E \leq X$ it holds that $B \leq 0$ and $C(B) = r_o B = r_o(E - X)$. The Hamiltonian is linear in E:

$$H = X(r_o - i) + \lambda_2(r_o X - D) + \lambda_1 F(S)$$
$$+ \{(1 + \lambda_2)[pqS - (c + a + r_o)] - \lambda_1 qS\}E,$$

and has the partial derivative

$$H_E = -\lambda_1 qS + (1 + \lambda_2)[pqS - (c + a + r_o)]. \tag{4.3}$$

(ii) For $E > X$ it holds that $B > 0$, which implies $C''(B) > 0$. Then, if $\lambda_2 > -1$, it holds that $H_{EE} = -(1 + \lambda_2)C''(B) < 0$. We shall show that $\lambda_2 > -1$ which means that for $E > X$ the Hamiltonian is strictly concave in E. The Hamiltonian is maximized by a unique E, E^* say, satisfying

$$H_E = -\lambda_1 qS + (1 + \lambda_2)[pqS - c - a - C'(E^* - X)] = 0. \tag{4.4}$$

(iii) The costate λ_1 is the shadow price of the stock of resource. We proceed under the reasonable hypothesis that $\lambda_1 > 0$: since S is a 'good stock', it is unlikely that λ_1 would turn out to be negative. Appendix 1 proves that $\lambda_1 > 0$ is satisfied in the optimal solution.

(iv) Define the Lagrangian, L, by $L(E, D, S, X, \lambda_1, \lambda_2, \mu, \nu) = H + \mu D + \nu E$, where $\mu(t)$ and $\nu(t)$ are piecewise continuous multiplier functions. Necessary optimality conditions include $L_D = -\lambda_2 + \mu = L_E = H_E + \nu = 0$; $\nu, \mu \geq 0$, $\nu E = \mu D = 0$. The costate λ_2 is the shadow price of the stock of equity X. Because of $L_D = 0$ it holds that $\lambda_2 \geq 0$. Thus, $\lambda_2 > -1$ which proves that for $E > X$ the Hamiltonian is strictly concave in E.

4.1. Characterization of the candidate paths

Path 1: $D = E = 0$.

This path has already been characterized in sufficient detail in Section 3.

Path 3a: $D = 0, \quad E > 0, \quad B \leq 0 \quad \leftrightarrow \quad E \leq X$.

$E > 0$ implies $\nu = 0$ and hence $H_E = 0$. Since $B \leq 0$, the derivative H_E, given by (4.3), is equal to zero. Path 3a is a singular path and is sustainable on a nonzero interval of time if the stationarity conditions $H_E = \mathrm{d}H_E/\mathrm{d}t = 0$ are satisfied for all t in the interval. Using the stationarity conditions as well as (2.2), (4.1), and (4.2), one finds that Path 3a is sustainable on a nonzero interval if $S(t)$ is kept constant at the level S_r. The level S_r is the constant, positive, and unique solution to

$$\phi(S) = F'(S) + \frac{[c + a + r_o]F(S)}{S[pqS - (c + a + r_o)]} - r_o = 0. \tag{4.5}$$

Consider (4.5), recall (2.1), and note that $\phi(S_m) = F'(S_m) - r_o < 0$. Define the positive constant M by $M := (c + a + r_o)/pq$. Then $H_E = 0$ in (4.3) implies $S > M$. It holds that $\phi(S) \to +\infty$ for $S \downarrow M$. For $S > M$ the derivative $\phi'(S)$ equals

$$F''(S) + \frac{(c + a + r_o)[pqS - (c + a + r_o)][SF'(S) - F(S)] - pqSF(S)}{S^2[pqS - (c + a + r_o)]^2}.$$

It holds that $\phi'(S) < 0$. This follows from concavity of F and $pqS - c - a - r_o > 0$. The latter is implied by $H_E = 0$ in (4.3). Thus, the level S_r exists and is the unique solution to (4.5) on the interval (M, S_m). The effort needed to sustain S_r is given by $E_r = F(S_r)/qS_r = \text{const.} > 0$.

Path 3b: $D = 0, \quad E > 0, \quad B > 0 \quad E > X.$

Differentiate in (4.4) totally with respect to time and use (2.3), (4.1), (4.2), and (4.4). At points at which B is differentiable, we obtain (3.2). Equity evolves according to

$$\dot{X} = [pqS - (c + a)]E - C(B).$$

It holds that $pqS - c - a - C'(B) > 0$ which is implied by (4.4) and the strict convexity of $C(B)$ for $B > 0$. Hence X is strictly increasing.

The level of debt B is never constant on Path 3b. To see this, assume $B = \text{const.} > 0$ on a positive interval. Then S given by (3.2) is constant. When the resource stock is constant, so is effort E. Using (2.6) and (2.7) implies that X is constant but this contradicts the fact that X is strictly increasing on Path 3b.

A full analytical characterization of trajectories $E(t)$, $S(t)$, and $B(t)$ on Path 3b seems inattainable. In a numerical example with a logistic growth function the trajectories behaved nicely: S increased (in a concave way) to S_i, and B increased in a concave way to B_i (and was larger than B_i on a short interval of time before T).

Path 2: $D > 0$, $E > 0$.

Since D is positive, it holds that $H_D = 0$. This implies that the costate λ_2, and its time derivative, must be zero on Path 2. Inserting into (4.2) yields $C'(B) = i$. Thus, B is positive and constant. Eq. (4.4) holds which means that (3.2) is valid. Combining (3.2) with $C'(B) = i$, and using the fact that B is constant, yields (3.1). Existence of the level S_i satisfying (3.1) can be proven in the same way as for the level S_r above. When S is constant, equal to S_i, E is constant, too, and equals $E_i = F(S_i)/qS_i$. When E and B both are constant, equity X is constant. Using this, and $pqS - c - a - C'(B) > 0$ (cf. Path 3b) shows that the dividend rate is constant, positive, and given by $D_i = (pqS_i - c - a)E_i - C(B_i)$.

Infeasible path

The case $E = 0$, $D > 0$ cannot occur on a nonzero interval of time. To see this, assume $D > 0$ on such an interval. Then $\mu = 0$, which implies that λ_2, and its time derivative, must be zero. Moreover, $i = C'(B)$, by (4.2). Then $B > 0$, using the definition of function $C(B)$. On the other hand, $E = 0$ implies $B < 0$, and we have a contradiction.

Our derivations show that state constraint $X \geq 0$ is satisfied along any of the candidate paths. Indeed, X is strictly increasing on Paths 1, 3a, and 3b. On Path 2, X is constant and positive. Since X_0 is positive, $X > 0$ holds for all t.

It remains to characterize the optimal investment policy along the above paths. At time 0, to start out on Path 1, an impulsive disinvestment is made to reduce the initial stock of capital, K_0, to zero. Investment is identically zero on Path 1. On Path 2, investment remains constant at the replacement level, that is, $I = aE_i > 0$. At the single instant of time, τ, at which Path 3a occurs, an impulsive (positive) investment is made to increase the effort from zero to the positive level $E^*(\tau^+) = X(\tau^+)$.

On Path 3b, we know the following. Define $f(t) = pqS - c - a - \lambda_1 qS/(1 + \lambda_2)$. Comparing $f = r_o$ [which holds on Path 3a] with $f = C'(B) = C'(E^* - X)$ [which holds on Path 3b] shows that $C'(B(\tau^+)) = r_o$ is required in order to have continuous costates and state S. Then $B(\tau^+) = 0$. Consider an initial interval, $(\tau, \tau + \delta)$ say, on Path 3b. Since $B(\tau^+) = 0$ and $B > 0$ must hold on Path 3b, it is clear that B must increase at least for δ sufficiently small. Hence, $B'(t) > 0$, $E'(t) > 0$, and $I(t) > 0$ for $t \in (\tau, \tau + \delta)$. Let T denote the instant of time at which Path 2 starts. Appendix 2 shows that $\dot{E}(T^-) > 0$ which implies $I(T^-) > 0$. Since E and its time derivative are continuous on Path 3b, $I(t)$ is also continuous. Hence, there exists on Path 3b a final interval of time, $(T - \varepsilon, T)$ say, during which $I(t)$ is positive.

Could there be disinvestment on the intermediate interval $(\tau + \delta, T - \varepsilon)$? Consider a scenario in which the threshold level S_{F0} is relatively close to S_i (e.g., because the zero-effort Path 1 has been applied for a long time). Then the initial stock of resource on Path 3b is relatively large and does not need to grow very much to reach the steady state level S_i. In such a case, relatively large harvesting rates may be warranted on Path 3b. To undertake such harvesting the firm needs capital. Then investment is positive during an initial interval but it may happen that the capital stock K is built up faster than what is needed for harvesting later on. Consider an instant, t' say, during the interval $(\tau + \delta, T - \varepsilon)$ and suppose that $K(t') > E(t')$. A disinvestment then should be made. Should the firm sell the redundant capital stock at once, at time t', or should it dispose of it gradually? We know that E and its time-derivative are continuous, and hence the investment rate is continuous. An impulsive sale of the redundant stock of capital is not possible and the firm has to sell off its redundant capital gradually. To sum up: in this specific scenario, an intermediate interval of disinvestment may, but does not necessarily, occur.

4.2. Synthesizing the paths

A path applied from any finite instant till infinity is called a terminating path. In view of the owners' objective, intuition suggests that the firm should not pay zero dividends over an infinite period of time.[13] Employing this argument, paths having $D(t) \equiv 0$ should not qualify as terminating paths. This rules out Paths 1, 3a, and 3b: Path 2 is the only terminating path since $D_i > 0$ on this path.

One could argue that the firm would rather like to start out by paying high dividends and then switch to a zero-dividend policy. Such behavior could be associated with a harvesting policy which drives the resource to extinction. To assess the argument, suppose that dividends must be positive on some initial interval of time. Then the firm must apply Path 2 as of time 0, since this path is the only path on which dividends are positive. Recall the assumption $S_0 < S_i$, where S_i is the steady state stock to be applied on Path 2. Obviously, S_0 cannot be increased impulsively from S_0 to S_i at time 0 and we conclude that the firm simply cannot start out on the 'dividend' Path 2. Hence, a policy of dividend payout on an initial interval of time is infeasible. Moreover, in our model, extinction never occurs, that is, $S(t) > 0$ for all t. On Path 1, S is strictly increasing from S_0, due to (2.2) and $E = 0$. On Path 3a, we have shown that $S > M = (c + a + r_o)/pq > 0$. Finally note that S has to be positive on Path 3b since if S became zero, it could not increase (cf. (2.2)) to reach $S_i > 0$ on Path 2.

Thus, let T denote the instant at which the terminal Path 2 starts, i.e., when the firm switches from a zero-dividend policy to the policy $D = D_i > 0$. The

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[13] See also Feichtinger and Hartl (1986, pp. 381–384), Van Hilten et al. (1993, p. 315).

optimal value of the firm's objective is

$$J^*(S_0, X_0) = \int_T^{\infty} D_i e^{-it} dt = e^{-iT} D_i/i, \tag{4.6}$$

which simply is the present value (at time 0) of the perpetuity D_i/i starting at time T. This instant depends on the initial point (S_0, X_0) as well as the effort policy employed on $t \in [0, T]$. For any fixed (S_0, X_0), (4.6) suggests that the firm chooses the sequence of paths which yields a minimal T. (Intuitively, the owners' wish to reach the dividend path, Path 2, as soon as possible). $T = 0$ would be a good choice but is infeasible unless either (S_0, X_0) happens to be equal to (S_i, X_i), or the point (S_i, X_i), through some impulsive adjustment, could be reached instantaneously from (S_0, X_0). The latter issue will be addressed below.

To obtain the optimal sequence of paths we apply a formal synthesizing procedure. It determines which path(s) can precede a given path, exploiting the continuity of states and costates and the necessary optimality conditions. Van Hilten et al. (1993) provide the details. The procedure starts out by identifying the set of feasible terminating paths. In our case this set is a singleton since it consists of Path 2 only.

The next step is to determine which path(s) can precede Path 2. The costate λ_2 is identically zero along Path 2. On Paths 1 and 3a we have $B \leq 0$, implying $C'(B) = r_o$ in (4.2). From $L_D = 0$ we know that $\lambda_2 \geq 0$ on Paths 1 and 3a. Using (4.2) shows that λ_2 is strictly increasing on these paths. Hence λ_2 can be zero only at the start of Path 1 or Path 3a. As λ_2 must be continuous, neither Path 1 nor Path 3a can precede Path 2; Path 3b is the only one that can precede Path 2.

The procedure continues by determining which path(s) can precede Path 3b, and so forth, and stops when no path can precede a given path, Path k say, and given that Path k satisfies the initial conditions. These calculations are mainly technical and have been relegated to Appendix 2. Here it suffices to report that the outcome is the following 'master string' of paths:

Path 1 $\rightarrow$ *Path 3a* $\rightarrow$ *Path 3b* $\rightarrow$ *Path 2.*

It is not always true that an optimal sequence should begin with Path 1. Depending on the initial conditions, a truncated version of the master string could be optimal. The firm would certainly wish to be on Path 2 as of time 0. In the case we have chosen to consider, $S_i > S_0$, this is impossible because S_0 cannot be increased impulsively to S_i. The same is true if the firm would wish to start out on Path 3a. Consequently, the firm can start out only on either Path 1 or Path 3b. Which of these two paths to apply at $t = 0$ depends on the initial conditions. To address this problem we need to characterize the string Path 1 $\rightarrow$ Path 3a $\rightarrow$ Path 3b $\rightarrow$ Path 2.

We distinguish two cases: $S_i > S_r$ and $S_i < S_r$, recalling that S_i is the resource stock associated with the terminating Path 2. S_r is the stock associated with the singular Path 3a. For the case $S_i > S_r$ the following result can be proved.

Lemma 1. *For $S_i > S_r$ it holds that Path 3a can only exist as a part of the string Path 1 → Path 3a → Path 3b → Path 2 → at one single instant of time.*

Proof. Let Δ be a nonnegative number. Assume that Path 3a can be sustained on an interval $[\tau, \tau + \Delta]$, $\Delta > 0$, $\tau > 0$, and $\tau + \Delta < T$. During this interval, $E = E_r$ and $S = S_r$. The state X increases on Paths 1, 3a, and 3b. Hence, $X(t) < X_i$ for all $t < T$. On Path 2, $E_i = X_i + B_i$, where $B_i > 0 \leftrightarrow X_i < E_i$. On Path 3a, $B(t) \leq 0$ and the following inequalities hold on the interval $[\tau, \tau + \Delta]$: $E_r = X(t) + B(t) \leq X(t) < X_i < E_i$. On the other hand, $E_r > E_i$ is implied by $S_i > S_r$ and the fact that $F(S)/qS$ is a decreasing function of S. By contradiction, $\Delta = 0$. This shows that Path 3a can only be applied at time τ. Q.E.D.

The lemma shows that, if $S_i > S_r$, the singular Path 3a cannot be sustained on a nonzero interval of time. This means that during such an interval, $H_E = 0$ in (4.3) cannot be true. Thus, for the linear part of the Hamiltonian, effort E is a strict bang–bang control: if $H_E < 0$ in (4.3), then put $E = 0$, if $H_E > 0$ in (4.3), then put $E = E^*$, E^* being the effort that satisfies (4.4). In what is to follow, we confine our interest to the case $S_i > S_r$. Later on we shall address the case $S_i < S_r$. Let us now establish the optimal trajectories.

Consider Path 3b and substitute $E = B + X$ into (2.2) and (2.11) (with $D = 0$, $K = E$). In connection with (3.2), this yields the three-dimensional, nonlinear differential equation system

$$\dot{S} = f(S, X, B), \quad \dot{X} = g(S, X, B), \quad \dot{B} = h(S, B).$$

Integrate this system in reverse time from time T, observing the initial conditions $S(T) = S_i$, $X(T) = X_i$, $B(T) = B_i$. Let τ be the first instant (in reverse time) at which $B = 0$. Noticing that $C'(0^+) = r_o$ and $C''(0^+) = 2k$, (3.2) yields

$$F'(S(\tau^+)) + \frac{F(S(\tau^+))(c + a + r_o)}{S(\tau^+)[pqS(\tau^+) - (c + a + r_o)]} - \frac{2k\dot{B}(\tau^+)}{pqS(\tau^+) - (c + a + r_o)} = r_o, \tag{4.7}$$

which defines the stock level S_{F0} of Proposition 1.[14] Thus, backwards integration on Path 3b determines an instant τ such that $S(\tau) = S_{F0}$,

[14] Comparing S_{F0} with S_r in yields $S_{F0} < S_r$. Moreover, $S_{F0} > M = (c + a + r_o)/pq$.

$B(\tau) = 0$. Moreover, it produces an $X(\tau) := X_\tau$. Recall that B has a discontinuity at $t = \tau$.

On Path 1 the equations of motion are simple

$$\dot{S} = F(S), \quad \dot{X} = r_o X, \quad \dot{B} = -r_o X.$$

Integrate this system in reverse time, starting from τ, with initial conditions S_{F0}, X_τ and $B(\tau^-) = -X_\tau < 0$. As S_0 is fixed, the time-to-go from S_{F0} to S_0 will be determined. Let z denote the instant at which $S = S_0$. (Hence $T = z + \tau$). Then we obtain $S(z) = S_0$ and a pair $(B(z), X(z))$ such that $B(z) = -X(z)$.

It would only be coincidental that the initial condition $X(z) = X_0$ is satisfied. However, if $X(z) < X_0$, an instantaneous distribution of dividends at time z can eliminate the difference. Such a distribution is financed by an impulsive reduction of the amount lent. (Recall that the entire capital stock K_0 is sold at time 0 when the firm starts out on Path 1.) On the other hand, if $X(z) > X_0$, an impulsive issue of new equity must be made (and is allowed by assumption) at time 0. These additional funds are used for an impulsive increase of the amount lent.

We are ready to start to prove Proposition 1. Using (4.3) and $L_E = 0$ shows that $S \leq M$ is sufficient for optimality of Path 1. Hence for all S_0 such that $S_0 \leq M$, the sequence Path 1 → Path 3a → Path 3b → Path 2 certainly is optimal.[15]

The next thing to show is that, for $S_0 \in (M, S_{F0})$, the sequence starting with Path 1 is optimal. For $t \in (0, \infty)$ the sequence of effort rates is $E^{\text{opt}} = (0, E^*(t), E_i)$. The associated stocks of resource and equity are $S^{\text{opt}}(t)$ and $X^{\text{opt}}(t)$, respectively, such that $S^{\text{opt}}(0) = S_0$, $X^{\text{opt}}(0) = X_0$, $S^{\text{opt}}(T) = S_i$, $X^{\text{opt}}(T) = X_i$. Now, for any fixed $t \in (0, T)$, consider the control problem having initial condition $(t, S^{\text{opt}}(t), X^{\text{opt}}(t))$ such that $M < S^{\text{opt}}(t) < S_{F0}$, $X^{\text{opt}}(t) < X_i$. Denote this problem by $\mathscr{P}$. By Bellman's principle of optimality, the restriction of E^{opt} to the interval (t, T) is a solution of $\mathscr{P}$. But the restriction of E^{opt} to (t, T) *is* E^{opt} itself. This follows from the fact that at time t, Path 1 is still employed in E^{opt}. Thus, for $S_0 \in M, S_{F0})$, the sequence starting out with Path 1 is optimal. This completes the proof of part (i) of the proposition. For part (ii) of the proposition, suppose that the sequence that starts out with Path 1 is applied. Then S is strictly increasing on some initial interval and cannot pass continuously into S_{F0} at time τ. Thus, the firm cannot start out on Path 1 and has no other choice than to apply Path 3b.

It remains to deal with the case $S_r > S_i$. The calculations in Appendix 2 remain valid but Lemma 1 does not hold, i.e., S_r may or may not be sustainable

[15] Alternatively, note that if $S_0 \leq M$, the sequence Path 3b → Path 2 cannot be applied. This is because $S > M$ on Path 3b (which follows from (4.4)).

on a nonzero interval of time. We proceed in another way. Assume that S_r can be sustained and let θ denote the time of coupling between Path 3a and Path 3b. When integrating the three-dimensional differential equation system on Path 3b, backwards from T, one needs to satisfy three initial conditions at time T, and $B(\theta^+) = 0$, but now also the condition $S(\theta^+) = S_r$. Fulfilling five requirements having only three constants of integration and time θ at one's disposal, is impossible. Thus S_r cannot be sustained which implies that Path 3a only occurs at a single instant of time, θ say, where Path 1 passes into Path 3b. At this instant it holds that $B(\theta^+) = 0$ and $S(\theta^+) = S_{\mathrm{F0}} < S_r$. We conclude that the proposition remains valid for $S_r > S_i$. The proof of Proposition 1 is now complete. Appendix 3 deals with sufficiency and uniqueness of the solution given in Proposition 1.

5. Concluding remarks

The paper is a first attempt to combine two different streams of research: optimal harvesting of a renewable resource, including investments in harvesting capacity (e.g., Clark et al., 1979), and optimal capital investments and finance (e.g., Van Hilten et al., 1993).

We studied cases in which the firm initially, in terms of equity, is 'small'. Moreover, the firm starts out with a 'small' stock of resource. It turned out that there is a threshold level of the resource stock under which harvesting is not profitable. During an initial interval of time, the actual resource stock is below the threshold and the firm should dispose of its productive capacity. Under the assumption of reversible investment, this can be done immediately.[16] During this initial phase, the firm invests all its equity in financial assets.

When the resource stock has become sufficiently large, in the sense that it reaches the threshold, all the firm's financial assets are sold. The proceeds are used to finance an instantaneous purchase of harvesting capacity: investment in harvesting capacity has now become economically rational. During the following phase, the firm invests in harvesting capacity and harvests at a rate which does not decrease the population. The firm borrows, as an additional means of finance, to pay for the harvesting operations and its investments. This phase continues until the stock of resource reaches its long-run equilibrium. After that, all variables become stationary.

We have introduced a number of assumptions and now discuss some of these.

[16] If investment were irreversible, the initial capital stock would decrease smoothly over time, due to depreciation, as in Clark et al. (1979) and Kennedy (1989).

5.1. Resource harvesting assumptions

The assumptions of constant unit price and operating cost are not unusual. The assumption of a constant price amounts to saying that the price obtained is independent of the size of the harvest landed. Thus, the firm is a price-taker. We could suppose that the firms acts in a competitive market where the firm's harvest is too insignificant to have any effect on the overall price level (Clark, 1976). The price-taker assumption does not necessarily conflict with the assumption of single ownership to the resource. We might think of the stock of resource as of many noninteracting stocks whose harvests together form a single world market for this fish. The constant unit operating cost comes from the linear operating cost function cE. A linear cost function may be defendable within a certain range of values of E, but not necessarily for all values of E. Clark (1976) discusses various extensions to nonlinear cost functions. Incorporating a maximum effort rate adds a nonlinearity to the model, although in a somewhat dramatic way. Alternatively, an operating cost being convex in effort could be introduced.

5.2. Financial assumptions

We disregarded the possibility of obtaining additional funds through issues of new shares – except at time 0. Excluding the option of new equity financing has some support in empirical analyses: Sinn (1987) states that new equity quite often is a marginal means of finance. Fishery companies may have reasonably good access to bank credit and quite a few fishery firms are family owned and managed. These could be reasons for not attracting new equity.

Following, among others, Steigum (1983), we excluded the possibility of lending and borrowing at the same time. Relaxing this assumption would yield the balance sheet $K + F = X + B$, where $F \geq 0$ is the stock of financial assets and $B \geq 0$ the level of debt. It would complicate the analysis while adding perhaps not very much financial substance.

The rate of interest earned on financial assets was constant. The assumption could be relaxed by assuming that the firm earns a higher interest rate per dollar invested in such assets as the total amount invested increases. This is sometimes the case for customers having substantial bank deposits. The interest payable when borrowing was assumed to depend on the absolute amount of debt, that is, $r = r(B)$. We also studied the case in which the interest rate is specified as $r = r(B/X)$ (cf. Steigum, 1983) and found the steady state resource stock to be lower, and steady state effort higher, than with interest function $r(B)$. These differences can be explained by realizing that in the model with interest rate function $r(B/X)$, the cost of borrowing depends on the value of B relative to X, rather than the absolute value of B. This indicates that the cost of borrowing can still be relatively low even if B is large as long as X is large, too. In this way, a higher rate of effort can be financed by debt, an option which is not economic in the case of an interest cost $r(B)$.

5.3. Capital investment assumptions

We assumed that any amount of positive or negative investment could be made instantaneously, that is, jump (or impulsive) increases or decreases in the capital stock can be made at a single instant of time. Allowing such jumps simplifies the analysis. Alternatively, one could introduce the constraint $I_L \leq I(t) \leq I_U$ in which the lower and upper bounds are finite. This may be more realistic but the analysis becomes considerably more complicated since one needs to consider corner solutions. Moreover, satisfaction of the crucial identity $E \equiv K$ may not always be guaranteed.

The assumption of disinvestment may not be warranted in specific cases and it would be necessary to introduce the irreversibility condition $I(t) \geq 0$, $\forall t \in [0, \infty)$. According to this condition, the disposal of unwanted capacity is impossible, except for exogenous decay. Alternatively, one could maintain the possibility of disposal of capacity, but such that scrapping would occur at a price below unity. Both approaches were used in Clark et al. (1979) who, on the other hand, employed rather extreme financial assumptions. Allowing for disinvestment, but at a price less than the acquisition price, does not change the solution obtained by Clark et al. very much. Two things deserve mentioning. First, a disinvestment region in the (S, K)-plane is added. If the initial state lies in this region, an initial impulsive disinvestment is made. This is analogous to our model. Second, a former short-run equilibrium resource stock (a singular solution) is no longer sustainable.

Incorporation of irreversible investment in our model would yield three state variables because it could no longer be assumed that $E(t)$ always equals its upper bound $K(t)$. Rather than pursuing this much more complicated modeling approach, we proceeded under the reversible investment hypothesis. We have seen that the optimal investment rate is nonnegative, with two exceptions. The first one is the impulsive disinvestment at time 0 which is intuitive in the scenario at hand. The second one is that disinvestment may occur under specific circumstances during an intermediate phase on Path 3b. Hence, the assumption of reversible investment does not seem overly restrictive.

Appendix 1: Proof of $\lambda_1 > 0$

Let τ be the instant at which Path 1, via Path 3a, passes into Path 3b. Let T be the instant at which Path 3b passes into Path 2. On Path 2 it is easily verified that λ_1 is constant and positive. Hence $\lambda_1(T) > 0$. On Path 3b, insert the optimal trajectories for $E(t)$, $S(t)$, and $\lambda_2(t)$ on the right-hand side of (4.1).

Denote the resulting function by $h(\lambda_1, t)$. Now, $h(0, t) = -(1 + \lambda_2)pqE < 0$ because λ_2 and E both are positive. Using a 'bouncing-off' result of ordinary differential equations (Seierstad and Sydsæter, 1987, p. 415), we conclude that $\lambda_1 > 0$ for $t \in (\tau, T]$. By continuity of costates, $\lambda_1(\tau) > 0$. On Path 1, $h(0, t) = 0$. Applying the same result once more shows that $\lambda_1 \geq 0$ for $t \in (0, \tau]$. In fact, the equality sign can be excluded; if λ_1 became zero at some instant of time during the interval $(0, \tau]$, λ_1 would remain zero and could never reach $\lambda_1(\tau) > 0$. Finally, $\lambda_1(0) > 0$ by the continuity of the costates. Q.E.D.

Appendix 2: Identifying the candidate sequences of paths

We start by characterizing the conditions that must hold at $t = T$ at which Path 3b passes into Path 2. Recall that $\lambda_2 \geq 0$ on Path 3b, $\lambda_2 = 0$ on Path 2. Since λ_2 must be continuous, $\lambda_2(T^-) = 0$ must hold. Moreover, $\dot{\lambda}_2(T^-) = 0$ is implied by the continuity of B at $t = T$. On Paths 2 and 3b, the Hamiltonian is strictly concave in E. Hence, an optimal $E(t)$ is continuous, in fact, continuously differentiable on these paths (Feichtinger and Hartl, 1986, p. 84). Using (2.2), and that E and S are continuous at $t = T$, yields $\dot{S}(T^-) = 0$. It holds that $\dot{B}(T^-) = 0$. This follows from (3.2), noting that $C''(B) > 0$, and using the continuity of $S(t)$ and $B(t)$ at $t = T$. We conclude that $S(t)$ and $B(t)$ pass continuously into S_i and B_i, respectively, at time T. On Path 3b we have $\lambda_2 \geq 0$ and, by continuity, $\dot{\lambda}_2(t) \leq 0$ for $t \in (T - \varepsilon, T)$, $\varepsilon > 0$. Suppose that $\dot{\lambda}_2(t) = 0$ on this interval. Then $i = C'(B(t)$, implying $B(t) = B_i$. But this cannot be true because we have proved that B is never constant on Path 3b. Hence, $\dot{\lambda}_2(t) < 0$ for $t \in (T - \varepsilon, T)$. This implies $i < C'(B(t))$ and, since $i = C'(B_i)$, we obtain $\dot{B}(t) < 0$ for $t \in (T - \varepsilon, T)$. On Path 3b, X increases such that $X(T^-) = X_i$. Thus, $\dot{X}(T^-) > 0$, and since $\dot{B}(T^-) = 0$, it follows that $\dot{E}(T^-) > 0$. Since $\dot{E}(T^+) = 0$, E has a kink at $t = T$ but is still continuous at $t = T$. At time T, dividends D jumps from zero to $D_i > 0$. This gives the trajectory $X(t)$ a kink at $t = T$ (but X is continuous at $t = T$).

Proceeding recursively, the following possibilities arise:

(*i*) Path 1 $\rightarrow$ Path 3b $\rightarrow$ Path 2

(*ii*) Path 2 $\rightarrow$ Path 3b $\rightarrow$ Path 2

(*iii*) Path 3a $\rightarrow$ Path 3b $\rightarrow$ Path 2

Re (*i*): At the instant of coupling, θ say, between Path 1 and Path 3b, E jumps from zero to $E^* > 0$. X increases on both paths and must be continuous when Path 1 passes into Path 3b. At time θ, B jumps from $B(\theta^-) < 0$ to $B(\theta^+) > 0$. An optimal E must maximize the Hamiltonian for all t, also at discontinuities of E. On Path 3b (and Path 2), (4.4) is a necessary condition. Hence

$-\lambda_1(\theta^+)qS(\theta^+) + (1+\lambda_2(\theta^+))(pqS(\theta^+) - c - a) = (1+\lambda_2(\theta^+))C'(B(\theta^+))$. On Path 1, $H_E < 0$ in (4.3) is a necessary (and sufficient) condition, that is, $-\lambda_1(\theta^-)qS(\theta^-) + (1+\lambda_2(\theta^-))(pqS(\theta^-) - c - a) < (1+\lambda_2(\theta^-))r_o$. Since S and both costates must be continuous at time θ, we get $C'(B(\theta^+)) < r_o$. This is, however, ruled out by the definition of function $C(B)$ and we conclude that Path 1 cannot precede Path 3b.

Re (*ii*): On Path 2, X_i = const. > 0. On Path 3b, X increases. If Path 2 were to precede Path 3b, X would start to increase (from X_i) as of the beginning of Path 3b. But then Path 2 cannot be coupled after Path 3b since such coupling would require $X = X_i$ at the instant at which Path 3b passes into the terminating Path 2. Thus, Path 2 cannot precede Path 3b.

Re (*iii*): This coupling is feasible. X increases on both paths. On Path 3a, $H_E = 0$ in (3.3) and (4.4) holds on Path 3b. At the time of coupling between Path 3a and Path 3b, τ_1 say, it must hold that $C'(B(\tau_1^+)) = r_o$ because S and both costates must be continuous. It follows that $B(\tau_1^+) = 0$. Since $B(\tau_1^-) < 0$, the variable B has a discontinuity at the instant of coupling. The control E jumps upward to $E^*(\tau_1^+) = X(\tau_1^+)$ at the coupling instant.

We have established the sequence Path 3a → Path 3b → Path 2. It remains to be demonstrated that Path 1 is the only path which can precede Path 3a, and that no path can precede Path 1.

(*iv*) Path 1 → Path 3a → Path 3b → Path 2

(*v*) Path 3b → Path 3a → Path 3b → Path 2

(*vi*) Path 2 → Path 3a → Path 3b → Path 2

Re (*iv*): This coupling is feasible. X increases on Paths 1 and 3. S increases on Path 1 and at the coupling instant between Path 1 and Path 3a, τ say, it passes continuously into some positive value. B decreases on Path 1; $B(\tau^-) = -X(\tau^-) < 0$. Effort is switched from $E = 0$ to a positive value at time τ.

Re (*v*): On Path 3b, $B > 0$; on Path 3a, $B \le 0$. Let θ denote the instant at which Path 3b passes into Path 3a. As in (*i*), consider the Hamiltonian maximization conditions just before and just after time θ. $H_E = 0$ in (4.4) yields: $-\lambda_1(\theta^-)qS(\theta^-) + (1+\lambda_2(\theta^-))(pqS(\theta^-) - c - a) = (1+\lambda_2(\theta^-))C'(B(\theta^-))$, and $H_E = 0$ in (4.3) yields: $-\lambda_1(\theta^+)qS(\theta^+) + (1+\lambda_2(\theta^+))(pqS(\theta^+) - c - a) = (1+\lambda_2(\theta^+))r_o$. Continuity requirements yield $C'(B(\theta^-)) = r_o$, implying $B(\theta^-) = 0$. Consider the string Path 3b → Path 3a → Path 3b. First, $B = 0$ must hold just before the start and just after the end of Path 3a. [The latter requirement comes from (*iii*).] Second, X increases on Path 3a, which follows from $H_E = 0$ in (4.3). These two observations show that E must be larger at the end than at the start of Path 3a. This contradicts the fact that E is constant on Path 3a. In conclusion, Path 3b cannot precede Path 3a.

Re (*vi*): Consider the string Path 2 → Path 3a → Path 3b → Path 2. Recall that $X = X_i =$ const. on Path 2, and that X increases on Paths 3a and 3b. Since X must be continuous for all t, this coupling is infeasible.

It remains to be checked which path(s) could precede Path 1.

(*vii*) Path 2 → Path 1 → Path 3a → Path 3b → Path 2

(*viii*) Path 3a → Path 1 → Path 3a → Path 3b → Path 2

(*ix*) Path 3b → Path 1 → Path 3a → Path 3b → Path 2

Re (*vii*): On Path 2, (4.4) holds with $C'(B) = i$. On Path 1, $H_E < 0$ in (4.3). By continuity of S and the costates, $i < r_o$ must hold if the coupling is to be feasible. Assumption 1, however, produces infeasibility.

Re (*viii*): Continuity of S at the instant of coupling between Path 1 and its successor Path 3a precludes the coupling of Path 3a before Path 1. The reason is that $S = S_r$ on Path 3a while S is strictly increasing on Path 1.

Re (*ix*): This coupling is infeasible. The argument is very much the same as the one used in (*i*). It leads to the condition $C'(B(\theta^-)) < r_o$ which is impossible to satisfy.

Appendix 3: Sufficiency and uniqueness

We employ a sufficiency theorem for infinite horizon optimal control problems (Feictinger and Hartl, 1986, Thm. 2.4). The theorem essentially requires two things. First, the maximized Hamiltonian must be concave in (S, X) for any t and any $(\lambda_1(t), \lambda_2(t))$. Second, a limiting transversality condition must be satisfied. The latter is guaranteed in our model as any feasible state trajectory $(S(t), X(t))$ must remain nonnegative, both costates are nonnegative, and it holds that $e^{-it}\lambda_1 S_i \to 0$, $e^{-it}\lambda_2 X_i \to 0$ as $t \to \infty$. Consider the integrand of J in Section 4 as well as the right-hand sides of (2.2) and (2.11). All three expressions are concave functions of the quadruple (E, D, S, X). This can be seen by using the criterion of principal minors, noticing that D enters linearly in (2.11) and is absent from (2.2) and J (in the form employed in Section 4). Moreover, the costates are nonnegative and the necessary conditions of the maximum principle are satisfied. Thus, the Hamiltonian is concave in (E, D, S, X), implying that the maximized Hamiltonian is concave in (S, X). Concavity in (S, X) is not strict and uniqueness of the optimal pair (S, X) does not follow automatically. Similarly, the Hamiltonian is concave, but not strictly concave in (E, D), and uniqueness of the optimal pair (E, D) is not immediately guaranteed. However, on Paths 1 and 2, respectively, both controls are unique since $E = D = 0$ on Path 1, and $E = E_i > 0$, $D = D_i > 0$ on Path 2. Moreover, on Path 3b, $D = 0$ and since $H_{EE} < 0$ on Path 3b, E is uniquely determined. Path 3a only exists at a single instant of time.

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